

Radial solutions for a quasilinear elliptic system of Schrödinger type

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Abstract

In this paper we analyze the existence of entire radially symmetric solutions for Schrodinger system type $\Delta_{p_i} u_i + h_i(r) |\nabla u_i|^{p_i-1} = a_i(r) f_i(u_1, \dots, u_d)$ for $i = 1, \dots, d$ on \mathbb{R}^N where $p_i > 1$, $d \in \{1, 2, 3, \dots\}$, h_i and a_i are nonnegative radial continuous functions and f_i are nonnegative increasing continuous functions on $[0, \infty)$.

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1 Introduction

In this article we continue to study the existence results for systems such as

$$\begin{cases} \Delta_{p_1} u_1 + h_1(r) |\nabla u_1|^{p_1-1} = a_1(r) f_1(u_1, \dots, u_d) & \text{in } \mathbb{R}^N, \\ \dots \\ \Delta_{p_d} u_d + h_d(r) |\nabla u_d|^{p_d-1} = a_d(r) f_d(u_1, \dots, u_d) & \text{in } \mathbb{R}^N, \end{cases} \quad (1.1)$$

where $r := |x| \geq 0$ denotes the Euclidean length of $x \in \mathbb{R}^N$, $N \geq 3$, $d \geq 1$ is integer, Δ_{p_j} ($j = 1, \dots, d$) is the p_j -Laplacian operator defined by

$$\Delta_{p_j} u := \operatorname{div}(|\nabla u|^{p_j-2} \nabla u), \quad 1 < p_j < \infty,$$

$h_j, a_j : [0, \infty) \rightarrow [0, \infty)$ are radial continuous functions and f_j satisfy the following hypotheses

(C1) $f_j : [0, \infty)^d \rightarrow [0, \infty)$ are continuous in all variables;

(C2) f_j are non-decreasing on $[0, \infty)^d$ in all variables.

Particular forms of the Schrodinger system type like (1.1) have been considered in [2]-[21] and references therein. The problem considered in our paper it is considered in a general form that includes various types of problems from applied sciences. For example, the time-independent Schrodinger equation

$$(h^2/2m)\Delta u = (V - E)u \quad (1.2)$$

where h is the Plank constant, m is the mass of a particle moving under the action of a force field described by the potential V whose wave function is u and the quantity E is the total energy of the particle (see the book [6]). Also, some particular classes of systems studied in our work are used in the description of several physical phenomena such as the propagation of pulses in birefringent optical fibers and Kerr-like photorefractive media (see the articles [1], [13]). Moreover, in the absence of nonlinear gradient term such problems appear in medical science (see [12]).

Several theoretical results are available in the literature for the problem of the form (1.1). Most of the studies are about the existence or the nonexistence of positive radial ones, because the applications have been concentrated in symmetry theory. In particular, our research is closely related to [5, 9, 18] where the authors have obtained some theoretical interesting results and the paper [11] where the applications can be found. In [18], the author consider the *Schrödinger* system type

$$\left\{ \begin{array}{l} \Delta_{p_1} u_1 = H_1(|x|) u_2^{\alpha_1}, \\ \Delta_{p_2} u_2 = H_2(|x|) u_3^{\alpha_2}, \\ \dots \\ \Delta_{p_m} u_m = H_m(|x|) u_{m+1}^{\alpha_m}, \quad u_{m+1} = u_1, \end{array} \right. \quad x \in \mathbb{R}^N, \quad (1.3)$$

where α_i and p_i ($i = 1, 2, \dots, m$) are constants satisfying

$$\alpha_1 \cdots \alpha_m > (p_1 - 1) \cdots (p_m - 1),$$

Δ_{p_i} ($i = 1, 2, \dots, m$) is the p_i -Laplacian operator and the functions H_i ($i = 1, 2, \dots, m$), are nonnegative continuous functions on $[0, \infty)$. Under these hypotheses and some additional conditions he established the existence and non-existence results for solutions of the system (1.3) which will be improved here. Our solving method gives a stronger meaning to the obtained solutions, compared with the famous book "Particle Physics and the Schrodinger Equation" [6] where similar solutions are detected.

Before we start to describe our results let us mention that the problem of existence of solutions to (1.1) has received an increased interest with Alan Lair's recent paper [9]. In [9] the author considered problem

$$\left\{ \begin{array}{l} \Delta u_1 = a_1(|x|) u_2^\alpha(|x|), \quad (\alpha \in (0, 1]), \\ \Delta u_2 = a_2(|x|) u_1^\beta(|x|), \quad (\beta \in (0, 1]), \end{array} \right. \quad x \in \mathbb{R}^N. \quad (1.4)$$

The main result achieved by Alan Lair may be summarized as follows:

Proposition 1.1. *Problem (1.4) has an explosive radial symmetric solution on \mathbb{R}^N if and only if continuous radially symmetric functions $a_{i=\overline{1,2}} : [0, \infty) \rightarrow [0, \infty)$ simultaneously meet the following conditions*

$$\begin{aligned} \int_0^\infty t a_1(t) \left(t^{2-N} \int_0^t s^{N-3} \int_0^s \tau a_2(\tau) d\tau ds \right)^\alpha dt &= \infty \\ \int_0^\infty t a_2(t) \left(t^{2-N} \int_0^t s^{N-3} \int_0^s \tau a_1(\tau) d\tau ds \right)^\beta dt &= \infty. \end{aligned}$$

Moreover, the author issues the following problem:

"It remains unknown whether an analogous result for the system

$$\begin{cases} \Delta u_1(|x|) = a_1(|x|) f_1(u_2(|x|)) \text{ for } x \in \mathbb{R}^N, \\ \Delta u_2(|x|) = a_2(|x|) f_2(u_1(|x|)) \text{ for } x \in \mathbb{R}^N, \end{cases} \quad (1.5)$$

where f_1 and f_2 meet, for example the Keller-Osserman [7, 14] condition

$$\int_1^\infty \left[\int_0^s f_i(t) dt \right]^{-1/2} ds = \infty, \quad i = \overline{1, 2}, \quad (1.6)$$

or the Ye and Zhou [21] condition

$$\int_1^\infty [f_i(t)]^{-1} dt = \infty, \quad i = \overline{1, 2}. \quad (1.7)$$

We mention that in the paper [4] the author established for the system (1.5) a necessary condition as well as a sufficient condition for a positive radial solution to be large under conditions of the (1.6) type.

Now let us give a detailed description of our results. Throughout this paper we use the notations

$$\begin{aligned} j &= 1, \dots, d, & H_j(r) &:= r^{N-1} e^{\int_0^r h_j(t) dt}, \\ A_j(\infty) &:= \lim_{r \rightarrow \infty} A_j(r), & A_j(r) &:= \int_0^r \left(\frac{1}{H_j(t)} \int_0^t H_j(s) a_j(s) ds \right)^{\frac{1}{p_j-1}} dt, \end{aligned}$$

and the Lair's ([10, p.211]) quantity

$$F(\infty) = \lim_{r \rightarrow \infty} F(r), \quad F(r) = \int_a^r \left(1 + \sum_{j=1}^d f_j(s) \right)^{\frac{1}{1 - \min\{p_1, \dots, p_d\}}} ds; \quad r \geq a > 0.$$

We see that

$$F'(r) = \left(1 + \sum_{j=1}^d f_j(r) \right)^{-\frac{1}{\min\{p_1, \dots, p_d\} - 1}} > 0 \text{ for all } r > a$$

and F has the inverse function F^{-1} on $[a, \infty)$.

We now give our main theorems.

Theorem 1.2. *Suppose that (C1)-(C2) hold and that*

$$(C3) \quad F(\infty) = \infty.$$

Then the system (1.1) possesses at least one positive radial solution (u_1, \dots, u_d) . If, in addition, $A_j(\infty) < \infty$ ($j = 1, \dots, d$), the positive radial solution (u_1, \dots, u_d) is bounded. On the other hand, if $A_j(\infty) = \infty$ the positive solution (u_1, \dots, u_d) is entire large solution, i.e.

$$\lim_{r \rightarrow \infty} u_1(r) = \dots = \lim_{r \rightarrow \infty} u_d(r) = \infty.$$

Theorem 1.3. *Assume that (C1)-(C2) hold and that*

$$(C4) \quad F(\infty) < \infty;$$

$$(C5) \quad A_j(\infty) < \infty \quad (j = 1, \dots, d);$$

(C6) there exists $\beta > \frac{a}{d}$ such that

$$\sum_{j=1}^d A_j(\infty) < F(\infty) - F(d\beta).$$

Then, the system (1.1) possesses at least one positive bounded radial solution (u_1, \dots, u_d) satisfying

$$\beta + f_j^{1/(p_j-1)}(\beta, \dots, \beta) A_j(r) \leq u_j(r) \leq F^{-1} \left(F(d\beta) + \sum_{j=1}^d A_j(r) \right).$$

Theorem 1.4. (i) Assume that $A_{i=\overline{1,d}}(\infty) = \infty$ and

$$\lim_{s \rightarrow \infty} \frac{\sum_{i=1}^d (1 + f_i(s, \dots, s))^{\frac{1}{\min\{p_1, \dots, p_d\}-1}}}{s} = 0. \quad (1.8)$$

Then the system (1.1) has infinitely many positive entire large solutions.

(ii) Furthermore, if $A_{i=\overline{1,d}}(\infty) < \infty$ and

$$\sup_{s \geq 0} \left[\sum_{i=1}^d (1 + f_i(s, \dots, s))^{\frac{1}{\min\{p_1, \dots, p_d\}-1}} \right] < \infty$$

then the system (1.1) has infinitely many positive entire bounded solutions.

2 Proof of Theorems

2.1 Proof of Theorem 1.2

We note that radial solutions of system (1.1) are solutions u_j ($j = 1, \dots, d$) to the ordinary differential system

$$\begin{cases} \frac{1}{r^{N-1}} \left(r^{N-1} |u'|^{p_1-2} u' \right)' + h_1(r) |u'_1|^{p_1-1} = a_1(r) f_1(u_1, \dots, u_d), \\ \dots \\ \frac{1}{r^{N-1}} \left(r^{N-1} |u'|^{p_d-2} u' \right)' + h_d(r) |u'_d|^{p_d-1} = a_d(r) f_d(u_1, \dots, u_d), \end{cases} \quad (2.1)$$

and that any solution (u_1, \dots, u_d) to the integral equations

$$\begin{cases} u_1(r) = \beta + \int_0^r \left(\frac{1}{H_1(t)} \int_0^t H_1(s) a_1(s) f_1(u_1(s), \dots, u_d(s)) ds \right)^{\frac{1}{p_1-1}} dt, \\ \dots \\ u_d(r) = \beta + \int_0^r \left(\frac{1}{H_d(t)} \int_0^t H_d(s) a_d(s) f_d(u_1(s), \dots, u_d(s)) ds \right)^{\frac{1}{p_d-1}} dt, \end{cases} \quad (2.2)$$

is a solution to (1.1).

We will begin by establishing a solution of (2.2) in $(C[0, R])^d$ for arbitrary $R > 0$. For this, we apply a standard iteration procedure by letting

$$u_1^0 = \dots = u_d^0 = \beta > 0$$

the central values for the integral equations system and generating a non-decreasing sequence $\{u_j^k\}_{1 \leq j \leq d}^{k \geq 1}$ in which u_j^k is calculated from u_j^{k-1} by

$$\begin{cases} u_1^k(r) = \beta + \int_0^r \left(\frac{1}{H_1(t)} \int_0^t H_1(s) a_1(s) f_1(u_1^{k-1}, \dots, u_d^{k-1}) ds \right)^{\frac{1}{p_1-1}} dt, \\ \dots \\ u_d^k(r) = \beta + \int_0^r \left(\frac{1}{H_d(t)} \int_0^t H_d(s) a_d(s) f_d(u_1^{k-1}, \dots, u_d^{k-1}) ds \right)^{\frac{1}{p_d-1}} dt. \end{cases} \quad (2.3)$$

Due to the form (2.3), for all $r \geq 0$, $j = \overline{1, d}$ and $k \in N$ we have $u_j^k(r) \geq \beta$. Furthermore, we can easy see that $\{u_j^k\}_{1 \leq j \leq d}^{k \geq 1}$ are non-decreasing sequence on $[0, \infty)$.

By conditions (C1) and (C2) we obtain

$$\begin{aligned} (u_1^k(r))' &= \left(\frac{1}{H_1(r)} \int_0^r H_1(s) a_1(s) f_1(u_1^{k-1}(s), \dots, u_d^{k-1}(s)) ds \right)^{\frac{1}{p_1-1}} \\ &\leq f_1^{\frac{1}{p_1-1}}(u_1^k(r), \dots, u_d^k(r)) A_1'(r) \leq \left(\sum_{j=1}^d f_j \left(\sum_{j=1}^d u_j^k(r) \right) \right)^{\frac{1}{p_1-1}} A_1'(r) \\ &\leq \left(1 + \sum_{j=1}^d f_j \left(\sum_{j=1}^d u_j^k(r) \right) \right)^{\frac{1}{\min\{p_1, \dots, p_d\}-1}} A_1'(r), \\ &\dots \\ (u_d^k(r))' &= \left(\frac{1}{H_d(r)} \int_0^r H_d(s) a_d(s) f_d(u_1^{k-1}(s), \dots, u_d^{k-1}(s)) ds \right)^{\frac{1}{p_d-1}} \\ &\leq f_d^{\frac{1}{p_d-1}}(u_1^k(r), \dots, u_d^k(r)) A_d'(r) \leq \left(\sum_{j=1}^d f_j \left(\sum_{j=1}^d u_j^k(r) \right) \right)^{\frac{1}{p_d-1}} A_d'(r) \\ &\leq \left(1 + \sum_{j=1}^d f_j \left(\sum_{j=1}^d u_j^k(r) \right) \right)^{\frac{1}{\min\{p_1, \dots, p_d\}-1}} A_d'(r). \end{aligned} \quad (2.4)$$

Summing up gives

$$\left(1 + \sum_{j=1}^d f_j \left(\sum_{j=1}^d u_j^k(t) \right) \right)^{-\frac{1}{\min\{p_1, \dots, p_d\}-1}} \cdot \left(\sum_{j=1}^d u_j^k(t) \right)' \leq \sum_{j=1}^d A_j'(t).$$

Integrating this over $[0, r]$, produces

$$\int_0^r \left(1 + \sum_{j=1}^d f_j \left(\sum_{j=1}^d u_j^k(t) \right) \right)^{-\frac{1}{\min\{p_1, \dots, p_d\}-1}} \cdot \left(\sum_{j=1}^d u_j^k(t) \right)' dt \leq \sum_{j=1}^d A_j(r)$$

for each $r > 0$, which can be rewritten as

$$\int_0^r F' \left(\sum_{j=1}^d u_j^k(t) \right) dt \leq \sum_{j=1}^d A_j(r) \text{ for each } r > 0,$$

from which we get

$$F \left(\sum_{j=1}^d u_j^k(r) \right) - F(d\beta) \leq \sum_{j=1}^d A_j(r) \text{ for all } r \geq 0. \quad (2.5)$$

Since F^{-1} is increasing on $[0, \infty)$, follows that

$$\sum_{j=1}^d u_j^k(r) \leq F^{-1} \left(F(d\beta) + \sum_{j=1}^d A_j(r) \right) \text{ for all } r \geq 0. \quad (2.6)$$

Since (C3) holds, we can see that

$$F^{-1}(\infty) = \infty. \quad (2.7)$$

It follows that the sequences $\left\{u_j^k\right\}_{1 \leq j \leq d}^{k \geq 1}$ are bounded and non-decreasing on $[0, R]$ for $R > 0$.

Thus

$$\left(u_1^k, \dots, u_d^k\right) \text{ converges to } (u_1, \dots, u_d) \text{ on } [0, R]^d. \quad (2.8)$$

Consequently (u_1, \dots, u_d) is the positive entire radial solution of system (1.1) in $\overline{B(0, R)} \subset \mathbb{R}^N$ with central values $u_1(0) = \dots = u_d(0) = \beta$.

Since R is arbitrary, we can use the diagonal argument to show (2.8) has a convergent subsequence on $(C[0, \infty))^d$ to a function denoted again by (u_1, \dots, u_d) and this is a solution of (1.1) in \mathbb{R}^N . A complete proof of this procedure can be found in the work of ([16]) where numerical results are also commented.

In addition, when

$$A_j(\infty) < \infty, \quad j = 1, \dots, d$$

we see by (2.6) that

$$\sum_{j=1}^d u_j(r) \leq F^{-1} \left(F(d\beta) + \sum_{j=1}^d A_j(\infty) \right) \text{ for all } r \geq 0$$

when

$$A_j(\infty) = \infty \text{ for } j = 1, \dots, d$$

by (C2) and the monotonicity of $\left\{u_j^k\right\}_{1 \leq j \leq d}^{k \geq 1}$ follows

$$u_j(r) \geq \beta + f_j^{1/(p_j-1)}(\beta, \dots, \beta) A_j(r), \text{ for all } r \geq 0 \text{ and } j = 1, \dots, d.$$

Then

$$\lim_{r \rightarrow \infty} u_1(r) = \dots = \lim_{r \rightarrow \infty} u_d(r) = \infty,$$

and our proof is complete.

2.2 Proof of Theorem 1.3.

In a manner similar to our Theorem 1.2 proof above, we obtain that

$$F \left(\sum_{j=1}^d u_j^k(r) \right) \leq F(d\beta) + \sum_{j=1}^d A_j(\infty) < F(\infty) < \infty. \quad (2.9)$$

Because F^{-1} is strictly increasing on $[0, \infty)$ we have

$$\sum_{j=1}^d u_j^k(r) \leq F^{-1} \left(F(d\beta) + \sum_{j=1}^d A_j(\infty) \right) < \infty \text{ for all } r \geq 0. \quad (2.10)$$

Moreover, since the sequence $\left\{u_j^k(r)\right\}$ is monotone it converges to some function $\{u_j(r)\}_{1 \leq j \leq d}$ on \mathbb{R}^N that in fact is a solution to (1.1) and the proof is complete.

2.3 Proof of Theorem 1.4

We first see that radial solutions of (1.1) are solutions (u_1, \dots, u_d) of the differential equations system

$$\begin{cases} (p_1 - 1) |u_1'(r)|^{p_1-2} u_1'' + \frac{N-1}{r} u_1'(r)^{p_1-1} + h_1(r) |u_1'(r)|^{p_1-1} = a_1(r) f_1(u_1(r), \dots, u_d(r)), \\ \dots \\ (p_d - 1) |u_d'(r)|^{p_d-2} u_d'' + \frac{N-1}{r} u_d'(r)^{p_d-1} + h_d(r) |u_d'(r)|^{p_d-1} = a_d(r) f_d(u_1(r), \dots, u_d(r)). \end{cases} \quad (2.11)$$

Since the radial solutions of (1.1) are solutions of the differential equations system (2.11) it follows that the radial solutions of (1.1) with

$$u_1(0) = \beta_1, \dots, u_d(0) = \beta_d$$

where β_i ($i = 1, \dots, d$) may be any non-negative numbers, satisfy:

$$\begin{cases} u_1(r) = \beta_1 + \int_0^r \left(\frac{1}{H_1(t)} \int_0^t H_1(s) a_1(s) f_1(u_1(s), \dots, u_d(s)) ds \right)^{1/(p_1-1)} dt, \\ \dots \\ u_d(r) = \beta_d + \int_0^r \left(\frac{1}{H_d(t)} \int_0^t H_d(s) a_d(s) f_d(u_1(s), \dots, u_d(s)) ds \right)^{1/(p_d-1)} dt. \end{cases} \quad (2.12)$$

Define

$$u_1^0 = \beta_1, \dots, u_d^0 = \beta_d \text{ for } r \geq 0.$$

Let $\{u_j^k\}_{j=1, \dots, d}^{k \geq 1}$ be a sequence of functions on $[0, \infty)$ given by

$$\begin{cases} u_1^{k+1}(r) = \beta_1 + \int_0^r \left(\frac{1}{H_1(t)} \int_0^t H_1(s) a_1(s) f_1(u_1^k(s), \dots, u_d^k(s)) ds \right)^{1/(p_1-1)} dt, \\ \dots \\ u_d^{k+1}(r) = \beta_d + \int_0^r \left(\frac{1}{H_d(t)} \int_0^t H_d(s) a_d(s) f_d(u_1^k(s), \dots, u_d^k(s)) ds \right)^{1/(p_d-1)} dt. \end{cases} \quad (2.13)$$

We remark that, for all $r \geq 0$, $j = 1, \dots, d$ and $k \in \mathbb{N}$

$$u_j^k(r) \geq \beta_j.$$

Moreover $\{u_j^k\}_{j=1, \dots, d}^{k \geq 1}$ are non-decreasing sequence on $[0, \infty)$ such that

$$u_i^k(r) \leq u_i^{k+1}(r) \leq \left[f_i \left(\sum_{j=1}^d u_j^k(r), \dots, \sum_{j=1}^d u_j^k(r) \right) \right]^{1/(p_i-1)} A_i(r), \quad i = 1, \dots, d. \quad (2.14)$$

Let $R > 0$ be arbitrary. It is easy to see that (2.14) implies

$$\sum_{i=1}^d u_i^k(R) \leq \sum_{i=1}^d \beta_i + \sum_{i=1}^d \left[1 + f_i \left(\sum_{j=1}^d u_j^k(R), \dots, \sum_{j=1}^d u_j^k(R) \right) \right]^{\frac{1}{\min\{p_1, \dots, p_d\}-1}} \sum_{i=1}^d A_i(R), \quad k \geq 1,$$

and so

$$1 \leq \frac{\sum_{i=1}^d \beta_i}{\sum_{i=1}^d u_i^k(R)} + \frac{\sum_{i=1}^d \left[1 + f_i \left(\sum_{j=1}^d u_j^k(R), \dots, \sum_{j=1}^d u_j^k(R) \right) \right]^{\frac{1}{\min\{p_1, \dots, p_d\}-1}} \sum_{i=1}^d A_i(R)}{\sum_{i=1}^d u_i^k(R)}, \quad k \geq 1. \quad (2.15)$$

Moreover, taking into account the monotonicity of

$$\left(\sum_{i=1}^d u_i^k(R) \right)_{k \geq 1},$$

there exists

$$L(R) := \lim_{k \rightarrow \infty} \sum_{i=1}^d u_i^k(R).$$

We prove that $L(R)$ is finite. Indeed, if not, we let $k \rightarrow \infty$, in (2.15) and the assumption (1.8) leads us to a contradiction. Since $u_i^k(R)$ are increasing functions, it follows that the map $L : (0, \infty) \rightarrow (0, \infty)$ is nondecreasing and

$$\sum_{i=1}^d u_i^k(r) \leq \sum_{i=1}^d u_i^k(R) \leq L(R), \forall r \in [0, R], \forall k \geq 1.$$

Thus the sequences $(u_i^k(R))_{i=1, \dots, d}^{k \geq 1}$ are bounded from above on bounded sets. We now define the following quantities

$$u_i(r) := \lim_{k \rightarrow \infty} u_i^k(r) \text{ for all } r \geq 0 \text{ and } i = 1, \dots, d.$$

Then u_i is a positive solution of (2.12).

Next, we show that u_i ($i = 1, \dots, d$), is a large solution of (2.12). Let us remark that by (2.13) we have the following estimate

$$u_i(r) \geq \beta_i + [f_i(\beta_1, \dots, \beta_d)]^{1/(p_i-1)} A_i(r), \text{ for all } r \geq 0 \text{ and } i = 1, \dots, d.$$

It follows from the assumption f_i are positive functions and $A_i(\infty) = \infty$, that u_i ($i = 1, \dots, d$) is a large solution of (2.12) and so u_i is a positive entire large solution of (1.1). Thus any large solution of (2.12) provides a positive entire large solution of (1.1) with $u_i(0) = \beta_i$. Since $\beta_i \in (0, \infty)$ ($i = 1, \dots, d$) was chosen arbitrarily, it follows that (1.1) has infinitely many positive entire large solutions.

(ii) Assume that

$$\sup_{s \geq 0} \left[\sum_{i=1}^d (1 + f_i(s, \dots, s))^{\frac{1}{\min\{p_1, \dots, p_d\} - 1}} \right] < \infty$$

holds, then by (2.15) we have

$$L(R) := \lim_{k \rightarrow \infty} \sum_{i=1}^d u_i^k(R) < \infty.$$

On the other hand

$$\sum_{i=1}^d u_i^k(r) \leq \sum_{i=1}^d u_i^k(R) \leq L(R), \forall r \in [0, R], \forall k \geq 1.$$

So the sequences $(u_i^k(R))_{i=1, \dots, d}^{k \geq 1}$ are bounded from above on bounded sets.

Let

$$u_i(r) := \lim_{k \rightarrow \infty} u_i^k(r) \text{ for all } r \geq 0 \text{ and } i = 1, \dots, d.$$

Then $u_i(r)$, ($i = 1, \dots, d$) is a positive solution of (2.12). It follows from (2.14) that $u_i(r)$, ($i = 1, \dots, d$) is bounded, which implies that (1.1) has infinitely many positive entire bounded solutions. This concludes the proof of Theorem 1.4.

Remark 2.1. If (C1), (C2), (C3) are satisfied then

$$\int_a^\infty \frac{ds}{f_j^{1/(\min\{p_1, \dots, p_d\}-1)}(s, \dots, s)} = \infty \text{ for all } j = \overline{1, d}.$$

Remark 2.2. (see [3]) If (C1)-(C2) and

$$\int_a^\infty \frac{ds}{f_j^{1/(\min\{p_1, \dots, p_d\}-1)}(s, \dots, s)} = \infty \text{ for all } j = \overline{1, d},$$

are satisfied, then

$$\int_a^\infty \frac{dt}{\left(\int_0^t f_j(s, \dots, s) ds\right)^{1/\min\{p_1, \dots, p_d\}}} = \infty \text{ for all } j = \overline{1, d}.$$

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